Indian J. Phys. **72B** (5), 505-514 (1998)

IJP B
an international journal

On flat plate collector-new approach

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Received 26 November 1997, accepted 15 May 1998

Abstract . The efficiency of a flat plate collector is estimated. The transfer of heat energy through its absorber plate considered as its main part of structure is studied by solving the heat diffusion equation using the Fourier series expansion technique. The temperature of the working fluid is also obtained considering the heat balance equation Computations on a simple suggested model is carried out as an illustrative example

Keywords Heat transfer, flat plate collector

PACS Nos. 44 10 +i, 44.30. +v

1. Introduction

The flat plate collectors are still among the most common devices for the solar heating systems. The performance of such collectors has aroused the interest of many investigators [1-14]. The main part of such a device is its absorber plate. It is shown that the efficiency of such a collector is affected by several factors, for example, it is a function of the absorber temperature and the absorber surface absorptivity [2,11,12], the convective heat loss and the reflection losses from the glass covers [13, 14], the kind of the working fluid, its mass flow rate, the variation of the incident solar radiation with time and with the angle of incidence [4, 8, 11].

The aim of the present trial is to study the flat plate collector to evaluate its efficiency in a general form considering the absorber plate to be thick such that a temperature gradient exists across its thickness.

2. Theory

A simple model for the flat plate collector is suggested. The thick absorber of dimensions L_x, L_y and thickness ℓ in meters. It is the main part of the collector and is taken as an upper surface of a container of dimensions L_x, L_y and L_z. The absorber is kept in a horizontal position and is exposed to the incident global solar radiation $q_o(t)$ (W/m^2). The sides are assumed to be perfectly thermally insulated, thus the problem is treated as one dimensional problem. The working fluid enters the container from one side of dimensions L_z L_y and flows along the y- direction with velocity V_y and emerges from the opposite side. The volumetric rate of flow is $G_y = L_z L_x V_y$ (m^3/s). The efficiency η of the collector as a measure of its performance is defined as :

$$\eta = [I_{output}] / [I_{input}] , \tag{1}$$

with,

$$I_{input} = \int_{\Delta t} A\, q_0\,(t)\, dt , \tag{2}$$

$$I_{output} = \int_{\Delta t} q\,(1,t)\, dt , \tag{3}$$

where A is the absorptivity of the absorber surface. The function $q_0\,(t)$ for the daily solar irradiance as a function of the local day time (t) for clear days in suggested [15] for $t_r = 0$, and $t_s = t_d$, to be in the form :

$$q_0(t) = q_{max}\ [16(t/t_d)^4 - 32(t/t_d)^3 + 16(t/t_d)^2] , \tag{4}$$

where q_{max} is the maximum value acquired at midday t_0, $t_d = (t_s - t_r)$ is the length of the day, taken as the difference between sunset t_s and sunrise t_r. $t_0 = [(tr + ts)/2]$, and t is the local day time. Inserting eq. (4) into eq. (2) one gets

$$I_{input} = \int A\, q_0\,(t)\, dt$$

$$= A\,[\,16/5\, q_{max}\, t_d (t/t_d)^5 - 8\, q_{max}\, t_d (t/t_d)^4 + 16/3\, q_{max}\, t_d (t/t_d)^3\ ]. \tag{5}$$

It is worth to note that the total daily energy flux (J/m^2) can be estimated as :

$$\phi = \int_0^{td} q_0(t)\, dt = 0.533\, q_{max}\, t_d \tag{6}$$

To get the function $q(\ell, t)$, the heat diffusion equation for the absorber is written in terms of the heat flux $q(z, t)$ rather than the temperature in the form :

$$[\,\partial\, q(z,t) / \partial\, t] = \alpha\, \nabla^2\ q(z,t), \quad t > 0,\ z \geq 0, \tag{7}$$

where $\alpha = (\lambda / \rho c_p)$ is the thermal diffusivity written in terms of the thermal conductivity λ and the heat capacity per unit volume (ρc_p).

Eq. (7) is subjected to the following initial condition :

$$\text{At}\ \ t = 0,\ \ q(z, 0) = 0 . \tag{8}$$

Two boundary conditions are also considered :

At $z = 0$,

$$q(0, t) = A q_0(t) = -\lambda\ [\ \partial T\,(z,\, t)/\partial z\ ]\ |\ z = 0 \tag{9}$$

and

$$\text{At}\ z = \ell,\ \ q(\ell, t) = hT\,(\ell, t) , \tag{10}$$

where $T(z, t)$ is the excess temperature relative to the ambient temperature and h (W/m^{2}K) is the heat transfer coefficient at the rear surface. Moreover, the heat balance equation that governs the problem, is written in the form :

$$\int^{t} A\, q_0(t)\, dt = \int^{\ell} \rho c_p\ \ T(z,t)\, dz + \int^{t} q(\ell,t) dt . \tag{11}$$

To solve eq. (7), let us transform it into a nonhomogeneous equation subjected to homogeneous boundary conditions on the space variable z [16, 17]. This can be realized by assuming that the required solution can be written as

$$q(z,t) = V(z,t) + W(z,t) \tag{12}$$

with

$$V(z,t) = Aq_0(t) + (z/\ell)\{ q(\ell,t) - A\, q_0(t)\} . \tag{13}$$

Substituting eq. (12) into eq. (7) and considering eq. (13), one gets a nonhomogeneous equation for $W(z,t)$ in the form ,

$$[\partial^2 W(z,t)/\partial z^2] - \{(1/\alpha)[\,\partial W(z,t)/\partial t]\} = F(z,t), \tag{14}$$

with $$F(z,t) = (1/\alpha)[\ A\dot{q}_0(t) + (z/\ell)\{\dot{q}(\ell,t) - A\dot{q}_0(t)\}], \tag{15}$$

where dot denotes differentiation with respect to time.

Eq. (14) is now subjected to the following conditions :

$$W(z,0) = 0, \tag{16}$$

$$W(0,t) = 0, \tag{17}$$

and $$W(\ell,t) = 0. \tag{18}$$

To solve eq. (14), Fourier series expansion technique [16] can be applied. Let us assume that

$$W(z,t) = \sum_{n=1}^{\infty} w_n(t) \sin(n\pi z/\ell) \tag{19}$$

and $$F(z,t) = \sum_{n=1}^{\infty} F_n(t) \sin(n\pi z/\ell). \tag{20}$$

The initial condition (16) gives :

$$W_n(0) = 0 . \tag{21}$$

From eq. (15) it can be concluded that

$$F_n(t) = (1/\alpha)\, A\, \dot{q}_0(t)\ . \tag{22}$$

Inserting eqs. (19) and (20) into eq. (14), one gets

$$\sum_{n=1}^{\infty} [(n\pi/\ell)^2 W_n(t) + (1/\alpha)\dot{W}_n(t) + F_n(t)] \sin(n\pi z/\ell) = 0 . \tag{23}$$

This gives

$$\dot{W}_n\ (t) + \alpha\ (n\pi\ /\ \ell)^2\ W_n(t) = -\alpha\ F_n(t)\ . \tag{24}$$

Eq. (24) has an integrating factor $\exp \int \alpha\ (n\pi/1)2\ dt.$.

Considering the initial condition, this gives

$$W_n(t) = exp(-\alpha\ (n\pi/\ell\ )^2\ t)\ [\ \int_0^t -\alpha\ F_n\ (t)\exp(+\alpha\ (n\pi/\ell)^2\ t)\ dt\ ] \tag{25}$$

The required solution for $q(z, t)$ is obtained in the following from :

$$q(z,t) = [Aq_0\ (t) + (z/\ell)\ [\ q(\ell,t) - Aq_0(t)] + \sum_{n=1}^{\infty} W_n(t)\sin(n\pi z/\ell)]. \tag{26}$$

The function $q(\ell, t)$ can be obtained using the relation

$$\int_{T(0,t)}^{T(z,t)} dT\ (z,t) = \int_0^z -[q(z,t)\ /\ \lambda\ ]\ dz \tag{27}$$

together with the heat balance eq. (11) and the relation [18] as

$$\sum_{n=1}^{\infty} (\cos kx/k) = \frac{1}{2}\ ln\ (1/2(1-\cos x)), \qquad 0 < x < 2\pi. \tag{28}$$

Finally, the following expression for $q(\ell, t)$ is obtained in the form .

$$\begin{aligned}
q(\ell,t\ ) &= [\ Aq_{max}\ /(\ell\ \rho c_p[(1/h)+(\ell/3\lambda)]]\ [\ (16/t_d^4\ )\ [(t^4/\gamma) - (4t^3/\gamma^2)\\
&+ (12t^2/\gamma^3) - (24t/\gamma^4) + (24/\gamma^5)\} - \{(384/t_d^4\gamma^5)\ e^{-\gamma t}\} - (32/t_d^3)\ \{\ (t^3/\gamma)\\
&- (3t^2/\gamma^2) + (6t/\gamma^3) - (6/\gamma^4)\} + \{(192/t_d^3\gamma^4)\ e^{-\gamma t}\} + (16/t_d^2)\ \{\ (t^2/\gamma) - (2t/\gamma^2)\\
&+ (2/\gamma^3)\} - \{(32/t_d^2\gamma^3)\ e^{-\gamma t}\}\ ] + \sum_{n=1}^{\infty} [(\ell\cos n\pi)\ Aq_{max}\ /(\lambda n\pi[(1/h)\\
&+ (1/3\lambda)\}\}]\ [(64/t_d^4)\ \ \{(t^3/\gamma) - 3t^2/\gamma^2) + (6t/\gamma^3) - (6/\gamma^4)\}\\
&+ \{(384/t_d^4\gamma^4)\ e^{-\gamma t}\} - (192/t_d^4\beta)\ \{\ (t^2/\gamma) - ((2t/\gamma^2) + (2/\gamma^3)\}\\
&- (384/t_d^4\beta\gamma^3)\ e^{-\gamma t}\} + (384/t_d^4\beta^2)\ \{\ (t/\gamma) - 1/\gamma^2\} + \{(384/t_d^4\beta^2\gamma^2)\ e^{-\gamma t}\}\\
&- \{(384/t_d^4\beta^3\gamma)\ (1-e^{-\gamma t})\} - \{(384/t_d^4\beta^3\ (\beta-\gamma))\}\ (e^{-\beta t} - e^{-\gamma t}\ )] - (96/t_d^3)\\
&\times \{\ (t^2/\gamma) - (2t/\gamma^2) + (2/\gamma^3)\} - \{(192/t_d^3\gamma^3)\ e^{-\gamma t}\} + (192/t_d^3\beta)\ \{\ (t/\gamma)\\
&- (1/\gamma^2)\} - \{(192/t_d^3\beta\gamma^3)\ e^{-\gamma t}\} - \{(192/t_d^3\beta^2\gamma)\ (1-e^{-\gamma t})\} + \{(192/t_d^3\beta^2\ (\beta-\gamma))\}\\
&\times (e^{-\beta t} - e^{-\gamma t})\ ] +\ (32/t_d^2)\ \{\ (t/\gamma) -\ (1/\gamma^2)\ ] + (32/t_d^2\gamma^2)\ e^{-\gamma t}\}
\end{aligned}$$

$$- \{(32/td^2\beta\gamma)(1-e^{-\gamma t})\} - \{(32/t_d^2\beta(\beta-\gamma)\}(e^{-\beta t}-e^{-\gamma t})] + [(0.0539689$$

$$\times \ell A q_{max})/\lambda\{(1/h)+(\ell/3\lambda)\}](64/t_d^4)\{[(t^3/\gamma)-(2t^2/\gamma^2)+(4t/\gamma^3)$$

$$-(4/\gamma^4)\} + \{(256/t_d^4\gamma^4)e^{-\gamma t}\} - (96/t_d^3)\{(t^2/\gamma)-(2t/\gamma^2)+(2/\gamma^3)\}$$

$$+\{(192/t_d^3\gamma^3)e^{-\gamma t})\} + (32/t_d^2)\{(t/\gamma)-(1/\gamma^2)+\{(32/t_d^2\gamma^2)e^{-\gamma t}\}]. \quad (29)$$

Substituting expression (29) for $q(1, t)$ into eq. (3) and neglecting terms of negligible computational values, one gets the required expression I_{out} in the form :

$$I_{out} = \int_0^t q(1,t)\,dt = A\,q_{max}\,[\{(16\,t_d/5)(t/t_d)^5 - (16/\gamma)(t/t_d)^4 + (64/\gamma^2 t_d)$$

$$\times (t/t_d)^3 - (192/\gamma^3 t_d^2)(t/t_d)^2 + (384/\gamma^4 t_d^3)(t/t_d) + (384/\gamma^5 t_d^4)(e^{-\gamma t}-1)\}$$

$$- 8td\,(t/t_d)^4 - (32/\gamma)(t/t_d)^3 + (96/\gamma^2 t_d)(t/t_d)^2 - (192/\gamma^3 t_d^2)(t/t_d)$$

$$-(192/\gamma^4 t_d^3)(e^{-\gamma t}-1)\} + \{(16t_d/3)(t/t_d)^3 - (16/\gamma)(t/t_d)^2 + 32/\gamma^2 t_d)$$

$$\times (t/t_d) + (32/\gamma^3 t_d^2)(e^{-\gamma t}-1)\}]. \quad (30)$$

The efficiency η can thus be obtained according to the definition given by eq (1).

3. The temperature of the working fluid

The average temperature of the working T_w over a certain time interval Δt can be defined as

$$\bar{T}_w(t) = (1/\Delta t)\int_0^t T_w(t)dt \quad (31)$$

The heat balance equation for the considered model over the same time interval can be written as :

$$\int_0^t q(1,t)dt = V\rho_w C_w\,[(1/\Delta t)\int_0^t T_w(t)dt] + \int_0^t G_v\rho_w C_w\,[(1/\Delta t)\int_0^t T_w(t)dt]dt, \quad (32)$$

$$\int_0^t q(1,t)dt = V\rho_w C_w \bar{T}_w(t) + \rho_w C_w \bar{T}_w(t)\int_0^t G_v(t)dt. \quad (33)$$

Hence, $$\bar{T}_w(t) = \int_0^t q(1,t)dt/[V\rho_w C_w + \rho_w C_w\int_0^t G_v(t)dt]. \quad (34)$$

where $V = Lx\,Ly\,Lz$, is the volume of the fluid in the container.

The left hand side of eq. (32) represents the heat energy delivered at the rear surface of the absorber over a time interval Δt.

The first term on the right hand side represents the heat energy stored during the time interval Δt by the fluid in the container.

The second term on the right hand side represents the heat energy gained by the fluid passing the container over the considered time interval.

For constant volumetric rate of flow, G_v = constant.

$$\bar{T}_w(t) = [\int^t q(1,t)dt / \{ L_x L_y L_z \rho_w C_w + \rho_w C_w G_v t \}]. \tag{35}$$

4. Computations

As an illustrative example, the efficiency for the considered model of the flat plate collector is computed considering the following values :

The thickness of the absorber $\ell = 10^{-2}$ m and its surface area $L_x L_y = 1m^2$. The third dimension of the container L_z $5*10^{-2}m$.

The material of absorber is copper. The physical parameters of copper are as follows [19].

$\rho_w = 8954$ kg/m^3, $C_{p_w} = 383.1$ J/kg.k, $\lambda = 386$ W/m.k and $\alpha = 1.234*10^{-5}$ m^2/s. The absorptivity is assumed to be unity *i.e* $A = 1$. For the incident global solar radiation the following parameters are considered [20].

$q_{max} = 915$ W/m^2, $t_d = 12.58$ hours. The physical parameters of water as the working fluid are taken as

$\rho_w = 1000$ kg/m^3, $C_{p_w} = 4.1818*10^3$ J/kg.k.

The coefficient of heat transfer $h = 5$ W/m^2.K, the function $q_0(t/td)$ is computed using eq.(4). The obtained results are given in Table 1 and are illustrated graphically in Figure 1. The volumetric rate of flow is assumed to be Gy = 5 × 10^{-7} m^3/sec. The obtained results for I_{input}

Table 1. The incident diurnal global solar irradiance q, as a function of the relative local day time (t/t_d)

(t/t_d)	$q(t/t_d)$
0	0
0 1	118 584
0.2	374.784
0 3	645 624
0 4	843 264
0 5	915 00
0 6	843.264
0 7	645 624
0 8	374.784
0.9	118.584
1.0	0

(eq. 5) are given in table 2, the results for I_{output} are given also in table II and both I_{input} and I_{output} are illustrated graphically in Figure 2 for comparison. The efficiency η is computed

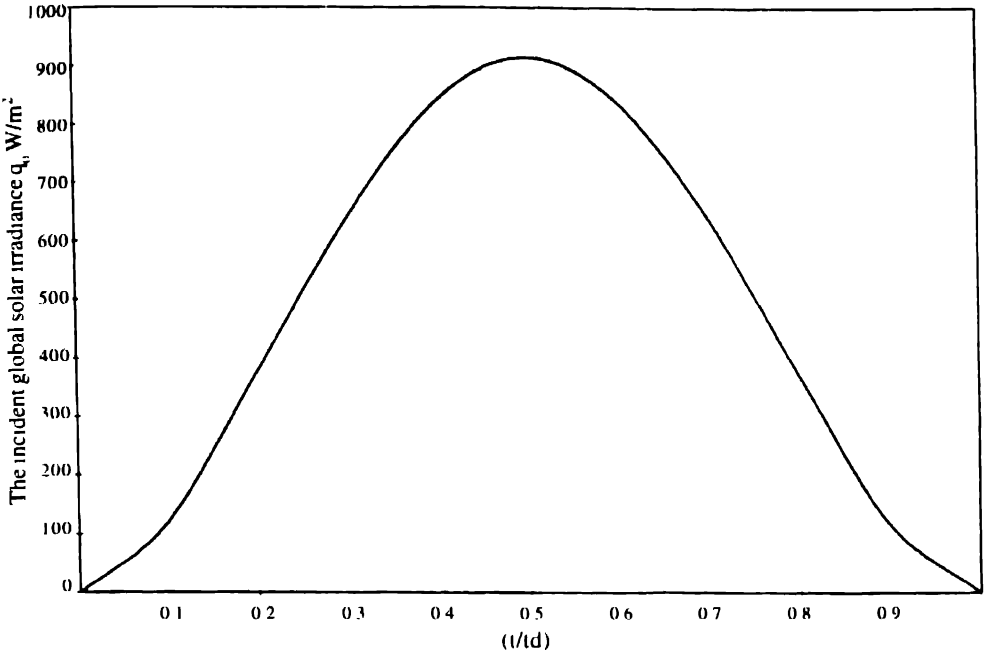

Figure 1. The incident global solar irradiance q_t (t) as a function of the relative local day time (t/t_d). q_{max} = 915 W/m², t_d = 12.58 hours [20]

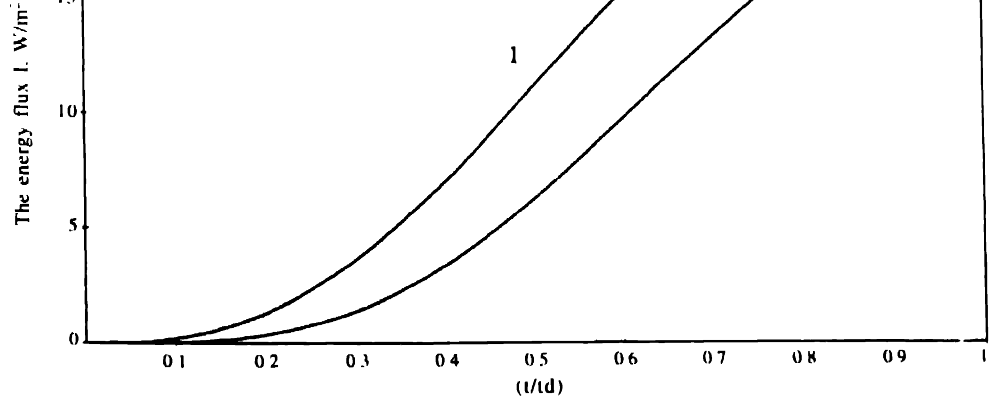

Figure 2. (1) The incident energy flux I_{input} as a function (t/t_d)
(2) The gain I_{output} as a function of (t/t_d).

according to eq. (1), the obtained results are given in Table 1 and are illustrated graphically in Figure 3. The average temperature of the working fluid is computed according to eq. (35). The obtained results are given in Table 2 and are illustrated graphically in Figure 4.

Table 2. The input energy flux, output flux, the efficiency η and the temperature of the working fluid T_w as a function of the relative local day time t/t_d.

(t/t_d)	I_{input} J/m^2*10^{-6}[eq (5)]	I_{output} J/m^2*10^{-6}[eq.(30)]	η% Eq (1)	T_w, k [eq. (35)]
0	0	0	0	0
. 0 1	0.189	0 035	18 5	0 16
0 2	1 280	0 358	28 0	1 57
0 3	3 604	1 380	38 3	5 81
0.4	7 016	3 337	47 6	13 51
0 5	11 050	6 187	56 0	24 13
0 6	15 085	9.636	63 9	36 24
0 7	18 500	13.224	71 5	48 02
0 8	20 820	16 448	79	57 74
0 9	21 911	18 903	86 3	64 23
1 0	22 101	20 431	92 4	67 26

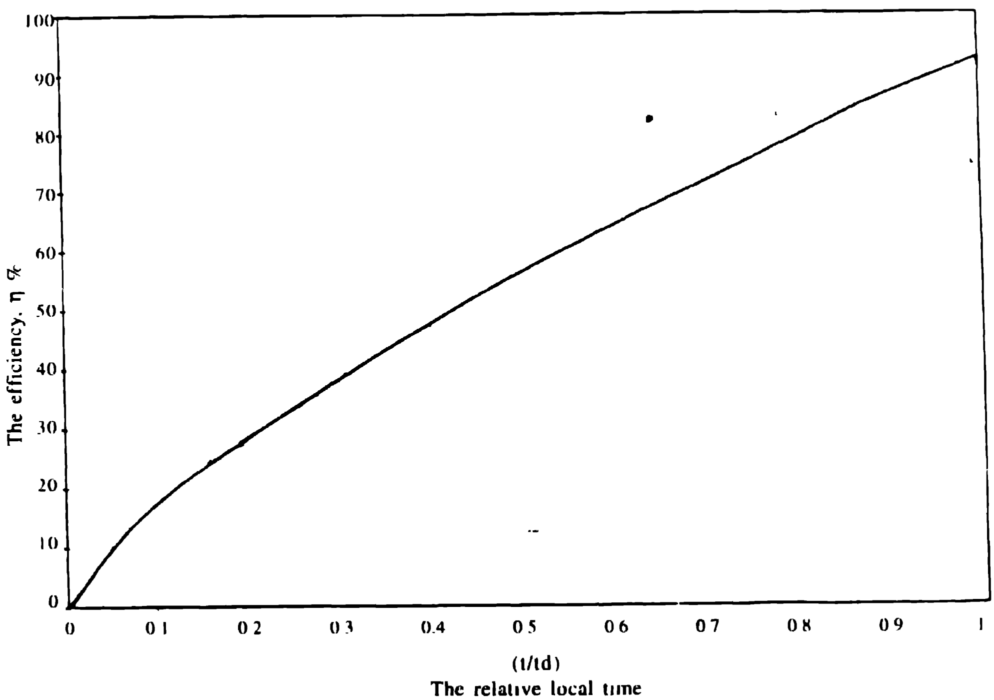

Figure 3. Efficiency η as a function of the relative local day time (t/t_d)

5. Conclusion

The method used to treat the problem seems to be promising and leads to the following results :

(i) The efficiency η is a function of the maximum value of the diurnal incident solar radiation q_{max}, the length of the solar daytime t_d, the surface absorptivity A, the thickness ℓ, and the thermal diffusivity $\alpha = (\lambda/\rho C_p)$ of the absorber plate. Moreover, it depends on the heat transfer coefficient h.

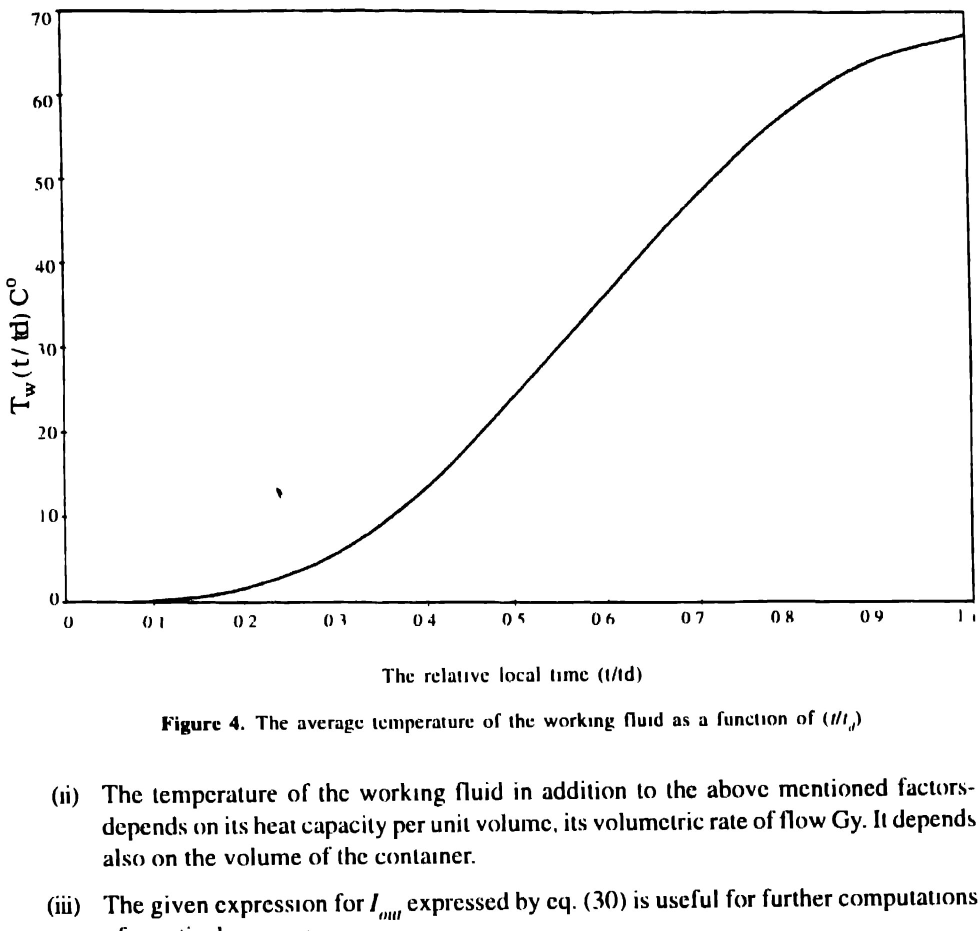

Figure 4. The average temperature of the working fluid as a function of (t/t_d)

(ii) The temperature of the working fluid in addition to the above mentioned factors-depends on its heat capacity per unit volume, its volumetric rate of flow Gy. It depends also on the volume of the container.

(iii) The given expression for I_{out} expressed by eq. (30) is useful for further computations of practical purposes.

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